Physics Notes

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Class:10+1
Unit: VI
Topic: Gravitation

SYLLABUS: UNIT-VI

Keplar's laws of planetary motion. The universal laws of gravitation.

Acceleration due to gravity and its variation with altitude and depth.

Gravitational potential energy; gravitational potential, Escape velocity, Orbital velocity of a satellite, Geo-stationary satellites.

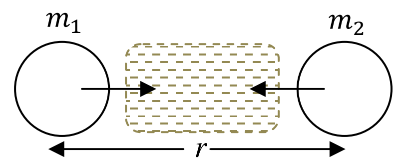
Q1. State and Explain Newton's law of gravitation in

a) Scalar form

b) **Vector** form

Ans a) **Newton's law in scalar form**:-

Force of attraction due to mass of body is termed as gravitational force



$F \propto m_1 m_2$

$\propto \frac{1}{r^2}$

$F \propto \frac{m_1 . m_2}{r^2}$

$$\boxed{F = \frac{G . m_1 . m_2}{r^2}}$$

Univ. Gravitation Constant 6.6 x 10^{-11} N $\frac{-m^2}{kg^2}$

It is called Universal Gravitational Constant because it does not depend upon material in between the two bodies.

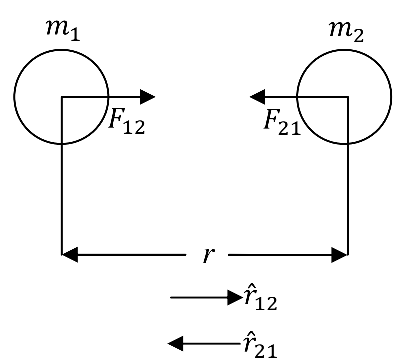
b) **Newton's Law in Vector Form**:-

$|\vec{F}_{12}| = |\vec{F}_{21}| = \frac{G . m_1 . m_2}{r^2}$

$\vec{F}_{12} = \frac{G . m_1 . m_2}{r^2} . \hat{r}_{12}$

$\vec{F}_{21} = \frac{G . m_1 . m_2}{r^2} . \hat{r}_{21}$

$$\boxed{|\vec{F}_{12}| = |\vec{F}_{21}|}$$

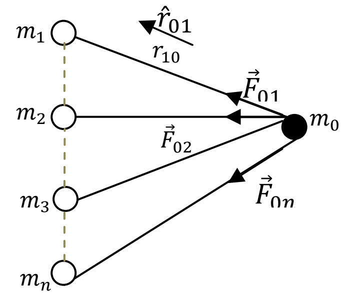


Characteristics:-

1. Independent of nature of medium.
2. Independent of the presence or absence of other bodies.
3. Independent of nature and size.
4. Action and reaction pair.
5. Central Forces.
6. Conservative Forces.

Q2. Explain superposition principle of force.

Ans. It states that the resultant gravitational force $\vec{F}$ acting on a particle due to number of point masses is equal to the vector sum of the forces exerted by the individual masses on the given particle.



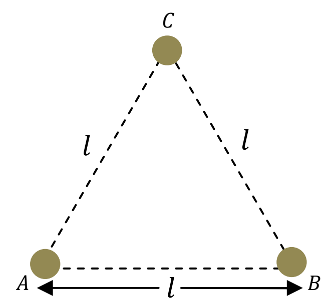
$\vec{F} \rightarrow$ net force on m_0

$\vec{F} \rightarrow \vec{F}_{01} + \vec{F}_{02}$ + ---------------- + $\vec{F}_{0n}$

$$= \frac{G.m_1 .m_0}{r_{10}^2} . \hat{r}_{01} + \frac{G.m_2 .m_0}{r_{20}^2} . \hat{r}_{02} + \text{-------------} + \frac{G.m_n .m_0}{r_{n0}^2} . \hat{r}_{0n}$$

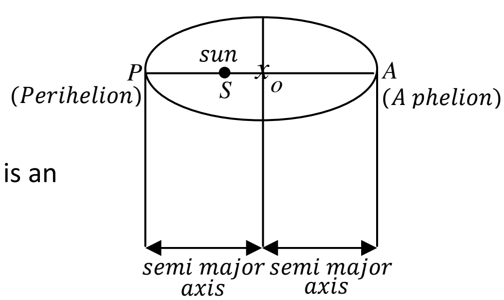
<u>Example</u>:

Three identical masses are placed at position A, B and C of a triangle as shown in Fig. Find resultant force on mass at C due to masses at A and B.



<u>Solution</u>:

Q3. State and Explain Kepler's I[st] Law.



Ans. 1[st] Law states that Every Planet revolves around the Sun is an Elliptical Orbit.

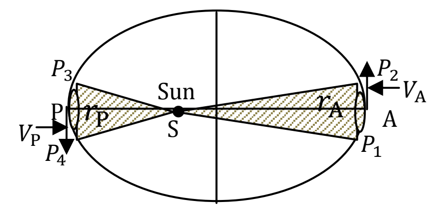
Q4. State and prove Kepler's 2nd Law.

Ans. **2nd Law**:-

The line joining a planet to the Sun sweeps out equal areas in equal interval of time.

OR

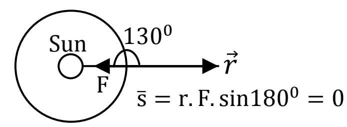
Areal Velocity of the planet around the Sun is constant.
As per Law of Conservation of Angular Momentum



L $= r.m.v \rightarrow$ constant

$r.m.v \rightarrow$ constant

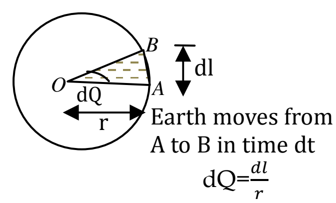
$\boxed{r.v \rightarrow \text{constant}}$



$\boxed{r_A.v_A = r_P.v_P}$

Deduction of Kepler's 2nd Law:-

Areal Velocity is constant



Small area OAB $= \frac{1}{2}.r\ .dl$

dA $= \frac{1}{2}.r\ .r.d\theta$

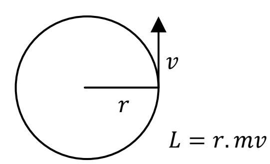
Dividing both sides by dt

$\frac{dA}{dt} = \frac{1}{2}.r^2.\frac{d\theta}{dt}$

$= \frac{1}{2}.r^2.\omega$

$= \frac{1}{2}.r(r.\omega)$

$= \frac{1}{2}.r.v$



Divide & Multiply the eq. by M

$= \frac{1}{2}(r.v\frac{M}{M})$

$\boxed{\text{Areal Vel.}, \frac{dA}{dt} = \frac{L}{2.m}}$

$$\left\{\begin{matrix} \tau = 0 \\ \frac{dL}{dt} = 0 \\ L \rightarrow Constant \end{matrix}\right\}$$

= constant

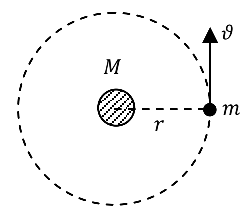
Q5. State and Explain Kepler's 3rd law.

Ans. **3rd Law**:-

The square of time period of revolution of a planet around the Sun is directly proportional to the cube of semi-major axis of its elliptical orbit.

$$\boxed{T^2 \propto R^3}$$

T→Time taken by the planet to go once around the Sun.
R→Semi-manor axis of the elliptical orbit.



Proof:

Time Period, $\boxed{T = \frac{2\pi . r}{\vartheta}}$ ——— 1

Also $\frac{m\vartheta^2}{r} = \frac{G.M.m}{r^2}$

$$\boxed{\vartheta = \sqrt{\frac{GM}{r}}} \text{ ——— } 2$$

Put value of ϑ from equation 2 in 1

$$T = \frac{2\pi . r}{\vartheta}$$

$$= \frac{2\pi . r}{\sqrt{\frac{GM}{r}}}$$

$$T = \frac{2\pi}{\sqrt{GM}} . r^{3/2}$$

Squaring both sides

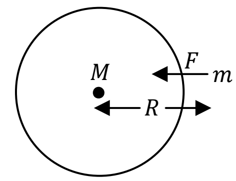
$$T^2 = \left(\frac{4.\pi^2}{G.M}\right) . r^3$$

$$\boxed{T^2 \quad \propto r^3}$$

Constant of proportionately is $\frac{4.\pi^2}{G.M}$

Q6. a) Define Gravity? units? Dimensions?
b) Compare Gravitation and Gravity?

Ans.a) M→Mass of Earth
m→Mass of small object on/near surface of Earth



$$F = \frac{G.M.m}{R^2}$$

"Gravity is the force of attraction exerted by earth towards its centre on a body lying on or near the surface of earth".

$$F = \left(\frac{G.M}{R^2}\right).m$$

$$= g.m$$

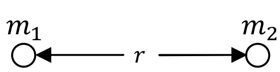
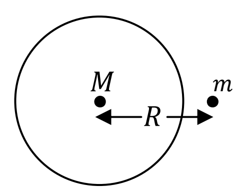
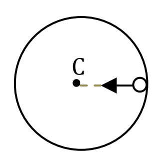
$$\boxed{F_{gravity} = m.g}$$

<u>NOTE</u>:- $F_{gravity} = \frac{G.m_1 M_2}{R^2}$

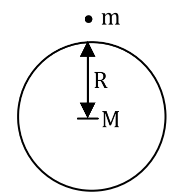
Units → N

Dimensions → $[M^1 L^1 T^{-2}]$

b)

Gravitation	Gravity
1.  $F = \frac{G.m_1 m_2}{R^2}$	1. $F = \frac{G.M.m}{R^2}$ 
2. Force between any two bodies.	2. Force on object on/near surface of earth.
3. Direction:-  Along the line joining the centres of two bodies.	3. Direction:-  Towards centre of earth.
4. F → 0 when r → ∞ $F = \frac{G.m_1 m_2}{\infty^2} = 0$	4. Gravity is "ZERO" at centre of earth. (To be proved)

Q7. Acceleration due to gravity? Discuss factors on which it depends?



Ans. $F_{gravity} = \frac{G.M.m}{R^2}$

"Force of gravity per unit mass is acceleration due to gravity".

$\left(\frac{F_{gravity}}{m}\right) = \frac{G.M}{R^2}$

$\boxed{g = \frac{G.M}{R^2}}$

It depends upon:-

a) *G*, a constant
b) *M*, mass of planet
c) *R*, Radius of planet (size)

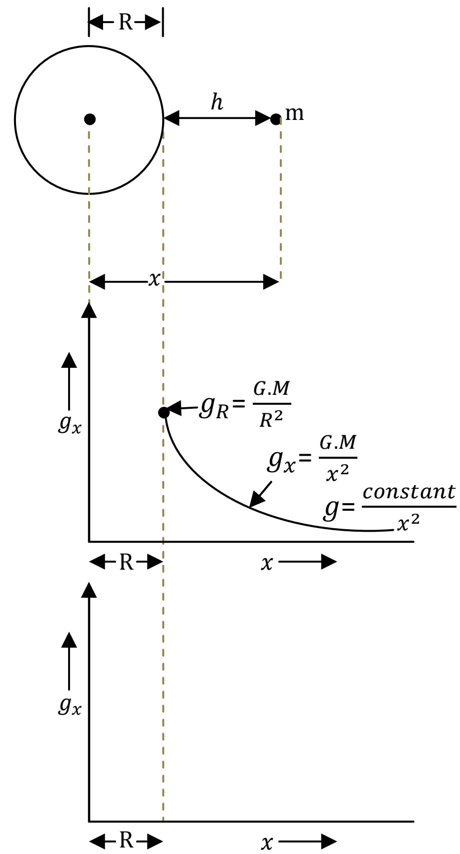
Q8. **For earth, discuss variation of *g inside and outside earth*.**

a) With height

b) With depth

Ans.**<u>Case I</u>**:- height **(R ≤ x < ∞)**

Let there be small mass at distance *x* from centre of earth.



$$F_x = \frac{G.M.m}{x^2}$$

$$\left(\frac{F_x}{m}\right) = \frac{G.M}{x^2}$$

$$\boxed{g_x = \frac{G.M}{x^2}}$$

In terms of height *h*

$$g_h = \frac{G.M}{(R+h)^2}$$

<u>Special Case</u>:- **h ≪ R**

$$g_h = \frac{G.M}{(R+h)^2}$$

$$= \text{G.M.}\ (R+h)^{-2}$$

$$= \text{G.M.}\ R^{-2}\left(1+\frac{h}{R}\right)^{-2}$$

$$= \frac{G.M}{R^2}\left(1+\frac{h}{R}\right)^{-2}$$

$$= \frac{G.M}{R^2}\left(1-\frac{2h}{R}\right)$$

$$g_n = \frac{G.M}{R^2}\left(1-\frac{2h}{R}\right)$$

$$g_n = g_s\left(1-\frac{2h}{R}\right)$$

$$\boxed{\frac{g_h}{g_{surface}} = \left(1-\frac{2h}{R}\right)}$$

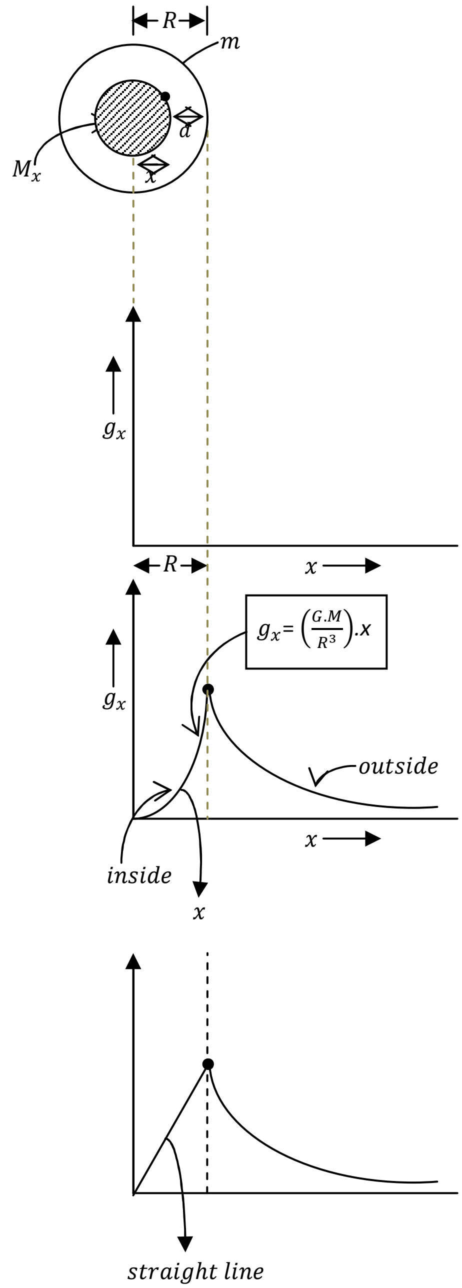
Case II:- depth **(0 ≤ x ≤ R)**

$F_x \quad = \frac{G.(M_x).m}{x^2}$

$\frac{F_x}{m} \quad = \frac{G.\left(\frac{4}{3}\pi x^3.\frac{M}{\frac{4}{3}\pi R^3}\right)}{x^2}$

$\boxed{g_x \quad = \left(\frac{G.M}{R^3}\right).x}$

Type y = constant (x) graph i.e. straight line



Special Case:-

1. x = 0

   $g_x = 0$

2. x = R

   $g_x = \left(\frac{G.M}{R^3}\right).R$

   $\boxed{g_x = \left(\frac{G.M}{R^2}\right)}$

In terms of depth

x = R – d

$g_x = \left(\frac{G.M}{R^3}\right).(R\text{-}d)$

$= g\left(\frac{R-d}{R}\right)$

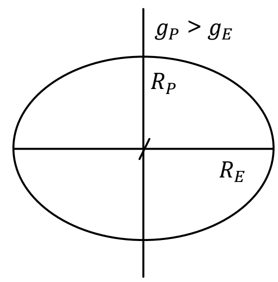
$\boxed{g_d = g\left(1-\frac{d}{R}\right)}$

Q9. Discuss variation of *g* due to shape of earth.

Ans. $g = \left(\frac{G.M}{R^2}\right)$

$R_E = R_P$

$R_E \rightarrow$ Radius of earth at equator
$R_P \rightarrow$ Radius of earth at poles



$g_E = \frac{G.M}{R_E^2}$

$g_P = \frac{G.M}{R_P^2}$

As $R_E > R_P$ So, $\boxed{g_E < g_P}$

Find %age increase in *g* on pole

$\frac{g_P}{g_E} = \frac{\frac{G.M}{R_P^2}}{\frac{G.M}{R_E^2}}$

$= \left(\frac{R_E}{R_P}\right)^2$

$= \left(\frac{R_P+21km}{R_P}\right)^2$

$= \left(1+\frac{21\ km}{6300km}\right)^2$

$\simeq \left(1+\frac{1}{300}\right)^2$

$= 1 + 2\left(\frac{1}{300}\right)$ $\quad [(1+x)^n = 1+n.x]$

$= 1 + \frac{2}{300}$

$\boxed{\frac{g_P}{g_E} = 1.006}$

$R_E > R_P$

$\boxed{R_E = R_P + 21\text{km}}$

6300 km
63 km $\rightarrow$ 1%
21 km $\rightarrow \frac{1}{3}$%
$\rightarrow$ 0.33%

g increases by 0.6%

<u>Alternate</u>:-

R decrease by 0.33%

g $\alpha\ R^{-2}$

$\frac{dg}{g} = -2\left(\frac{dR}{R}\right)$

$= -2(-0.33)$

$= 0.66\%$

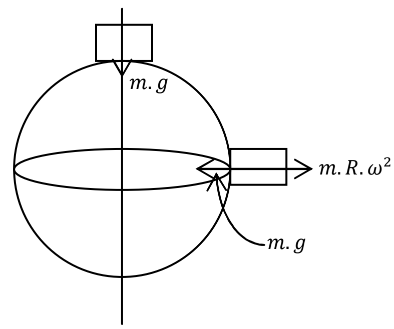
<u>Error Formula</u>

$y = x^n$

$\frac{dy}{y} = n.\left(\frac{dx}{x}\right)$

Q10. Discuss variation of *g* due to rotation of earth.

Ans.**Case I**:- on equator



$$g_E = \frac{m.g - m.R.\omega^2}{m}$$

$$= g - R.\omega^2$$

$$= \text{g-(6400Km)} \left(\frac{2\pi\ rad}{24\ hr}\right)^2$$

$$= \text{g-(6400 x 1000)} \left(\frac{2\pi\ rad}{24\ x\ 3600sec}\right)^2$$

= g-(0.03)

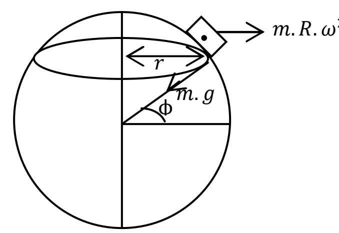
= 10-(0.03)

$\downarrow \qquad \downarrow$

100 0.3%

Case II:- No effect of rotation on poles

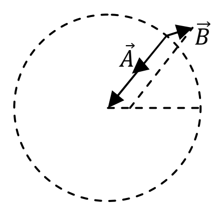
Reason:- Point on pole is not part of circular motion.



Case III:- Any point *P*, $\vec{g}_{eff}$ is resultant of two vectors at $\vec{A}$ and $\vec{B}$

where $|\vec{A}|$ = m.g

$|\vec{B}|$ $= m.R.\omega^2$



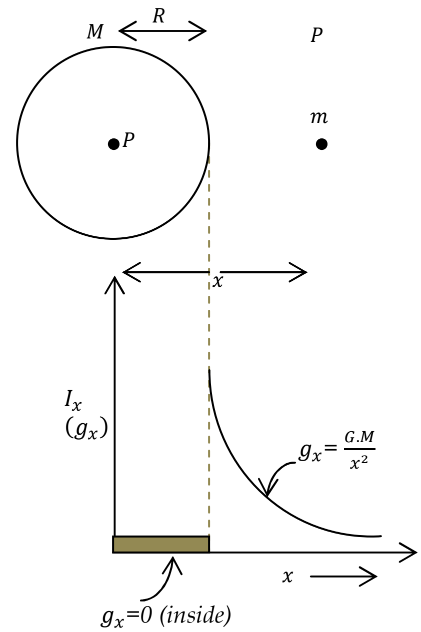
Angle between $\vec{A}$ and $\vec{B}$ depends on ϕ, to be given by examiner.

Q11. **Plot variation of Gravitational Field intensity**

a) Inside

b) Outside

A hollow spherical shell of mass *M* and radius *R*.



Ans.**<u>Case I</u>**:- **Outside shell (R < x < ∞)**

Point *P* is at distance *x*, small mass *m* at point *P*.

$$F = \frac{G.M.m}{x^2}$$

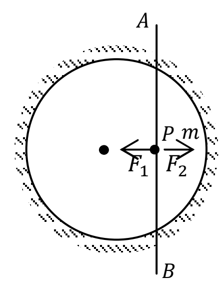
$$\left(\frac{F}{m}\right) = \frac{G.M}{x^2}$$

$\frac{F_x}{m}$ or $\boxed{g_x = \frac{G.M}{x^2}}$ i.e. $g_x \propto \frac{1}{x^2}$

<u>Case II</u>:- **Inside shell (0 < x < R)**

Mass to the left of line *AB* attracts *m* towards left with Force F_1.

Mass to the right of line *AB* attracts *m* towards right with Force F_2.



It can be proved $\boxed{F_1 = F_2}$

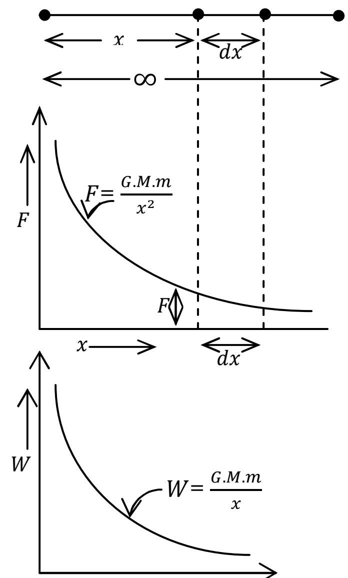
So, $\boxed{F_{net} = 0}$

So, $\boxed{g_x = 0}$ (inside)

Q12. a) Define Gravitational *P.E.*? Expression for *P.E.* for point mass *M*?

b) Derive expression of Gravitational *P.E.* for Earth (outside).

Ans.a) Amount of work to be done in taking mass m from x to ∞.



(a)

dw = F.dx

w $= \int dw$

$= \int F.dx$

$= \int \frac{G.M.m}{x^2}.dx$

$= \text{G.M.m} \left|\frac{x^{-2+1}}{-2+1}\right| \begin{matrix} x = \infty \\ x = x \end{matrix}$

$= -\text{ G.M.m} \left|\frac{x^{-1}}{-1}\right| \begin{matrix} x = \infty \\ x = x \end{matrix}$

$= -\text{ G.M.m} \left|\frac{1}{\infty} - \frac{1}{x}\right|$

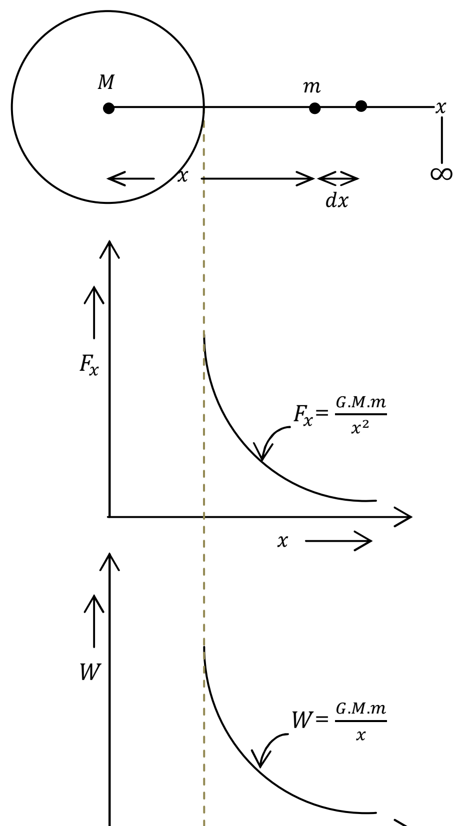
$$\boxed{\text{w} = \frac{\text{G.M.m}}{x}}$$

G.P.E,U = -work

$$\boxed{\text{U} = \frac{-\text{G.M.m}}{x}}$$

b) Gravitational Potential Energy for earth

Force between M and m



(b)

$F_x = \frac{G.M.m}{x^2}$

dw = F.dx

w $= \int dw$

$= \int \frac{G.M.m}{x^2}.dx$

$= \text{G.M.m} \int x^{-2}.dx$

$$\boxed{\text{w} = \frac{\text{G.M.m}}{x}}$$

$$\boxed{\text{G.P.E, U} = \frac{-\text{G.M.m}}{x}}$$

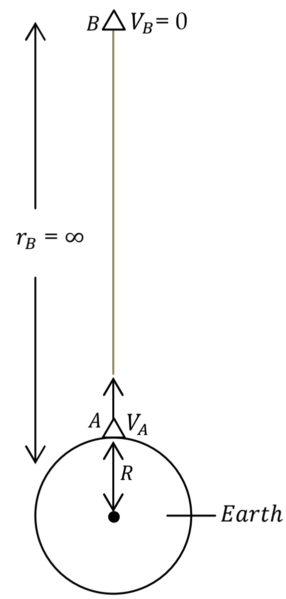
Q13. Derive an Expression for '*Escape Velocity*' from a planet?

Ans. **<u>Escape Velocity</u>**:-

Minimum speed with which the body has to be projected vertically upwards from the surface of earth so that it just crosses the gravitational field of earth and never return on its own.

<u>Proof</u>:-

Let the particle fired from surface of earth with speed V_A. It reached to point *B* where its speed is '*ZERO*' and *B* is far away from gravitation field.



As particle moves from position *A* to position *B*, Kinetic Energy decreases, Gravitational Potential Energy increase, Total Energy remains constant.

<u>As per Law of Conservation of Energy</u>:-

$$T.E_A = T.E_B$$

$$K.E_A + P.E_A = K.E_B + P.E_B$$

$$\frac{1}{2}.m.V_A^2 + \left(\frac{-GMm}{R}\right) = 0 + \left(\frac{-GMm}{\infty}\right)$$

$$\frac{1}{2}.m.V_A^2 + \left(\frac{-GMm}{R}\right) = 0 + 0$$

$$\frac{1}{2}.\cancel{m}.V_A^2 = \frac{GM\cancel{m}}{R}$$

$$V_A^2 = \frac{2GM}{R}$$

$$V_A = \sqrt{\frac{2GM}{R}} \qquad \boxed{g = \frac{GM}{R^2}}$$

$$= \sqrt{\frac{2.g.R^2}{R}}$$

$$\boxed{V_A = \sqrt{2.g.R}}$$

<u>For Earth</u>:- V $= \sqrt{2 \text{ x } 10 \text{ x } 6.4 \text{ x } 10^6}$

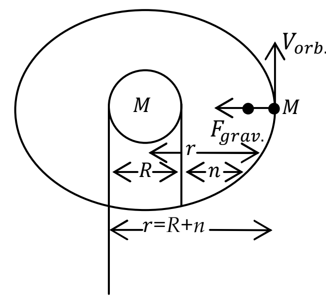
$= 11.2 \text{ x } 10^3$ m/sec

$$\boxed{\text{V} = 11.2\text{Km/sec}}$$

Q14. Derive an expression for

a) Orbital Speed
b) Time Period
c) Kinetic Energy
d) Potential Energy
e) T.E and significance of 'Binding Energy' for a satellite.

Ans. A particle of Mass *m* moves around planet of Mass *M* with Radius *r*.



a) **Orbital Speed**:-

'Minimum speed required to put the satellite into a given orbit around earth'.

$V_{orb} \rightarrow$?

$F_{grav.} = \frac{G.M.m}{r^2}$ ------------------ (1)

$F_C = \frac{m.V_{orb.}^2}{r}$ ------------------- (2)

$\frac{\cancel{m}.V_{orb.}^2}{\cancel{r}} = \frac{G.M.\cancel{m}}{\cancel{r}^2}$

$$\boxed{V_{orb} = \sqrt{\frac{GM}{r}}}$$

For a particle near surface of earth $r = R$

$V_{orb} = \sqrt{\frac{GM}{r}}$

$= \sqrt{\frac{g\cancel{R}^2}{\cancel{R}}}$

$$\boxed{V_{orb} = \sqrt{g.R}}$$

For Earth:- V $= \sqrt{10 \text{ x } 6.4 \text{ x } 10^6}$

$= 8 \text{ x } 10^3$ m/sec

= 8Km/sec

b) **Time Period**:- T $= \frac{2.\pi.r}{V_{orb}}$

$= \frac{2.\pi.r}{\sqrt{\frac{GM}{r}}}$

$$\boxed{T = \frac{2\pi}{\sqrt{GM}} . r^{3/2}}$$

Squaring both sides

$T^2 \; \alpha \; r^3$

c) **Kinetic Energy**:-

K.E. $= \frac{1}{2}.\text{m}.V_{orb.}^2$

$= \frac{1}{2}.\text{m}. \frac{G.M}{2r}$

$$\boxed{\text{K.E.} = \frac{G.M.m}{2r}}$$

d) **Potential Energy**:-

$$\boxed{\text{P.E.} = \frac{-G.M.m}{r}}$$

e) **Total Enery**:-

T.E. = K.E. + P.E.

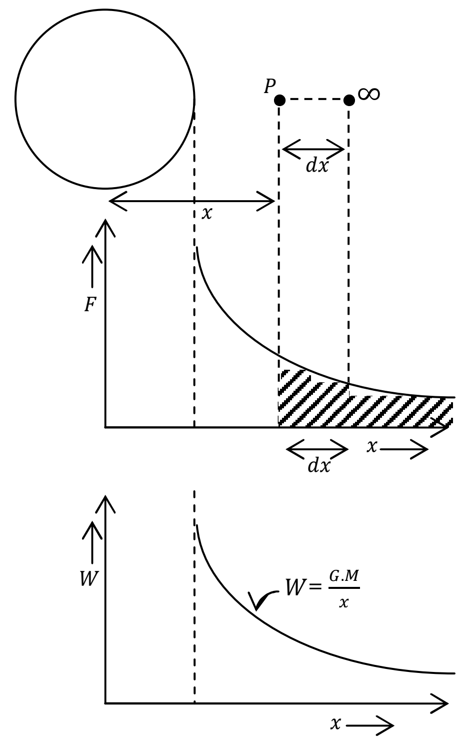
$= \frac{G.M.m}{2r} + \left(\frac{-G.M.m}{r}\right)$

$$\boxed{\text{T.E.} = \frac{-G.M.m}{2r}}$$

Q15. What is Gravitational Potential? Derive an expression for the same?

Ans. **Gravitational Potential**:-

The amount of work done in bringing a body of unit mass from infinity to that point without acc.



Units:- SI unit $= \frac{J}{Kg}$

Dimension:- $[V] = \frac{[work]}{[mass]}$

$= \frac{M^1L^2T^{-2}}{M}$

$$\boxed{[V] = M^0L^2T^{-2}}$$

Derivation:-

Work done in moving from P to ∞, W

W $= \int F.dx$

$= \int_x^\infty \frac{G.M.1}{x^2}.dx$

$=$ G.M $\int x^{-2}.dx$

$=$ G.M $\left[\frac{x^{-1}}{-1}\right]_{x=x}^{x=\infty}$

$=$ - G.M $\left[\frac{1}{x}\right]_{x=x}^{x=\infty}$ $\left\{\int x^n.dx = \frac{x^{n+1}}{n+1}\right\}$

$=$ - G.M $\left[\frac{1}{\infty} - \frac{1}{x}\right]$

$$\boxed{W = \frac{GM}{x}}$$

V_x $=$ - W

$= -\frac{GM}{x}$

$$\boxed{\text{Grav.Potential, } V_x = -\frac{GM}{x}}$$

Q16. Relation between Gravitational Force and Gravitational Potential Energy?

Ans. F $= \frac{G.M.m}{r^2}$

U $= \frac{-G.M.m}{r}$

Prove F = $\frac{-dU}{dr}$

R.H.S. $= \frac{-dU}{dr}$

$= \frac{-d}{dr}\left(\frac{-G.M.m}{r}\right)$

$= G.M.m\,\frac{d}{dr}\left(\frac{1}{r}\right)$

$= G.M.m\,(-1r^{-2})$

$= -\left(\frac{G.M.m}{r^2}\right)$

-ve sign shows that Force on mass *m* acts radically inwards towards *M* i.e. the Force of attraction nature.